

Complete higher dimensional global embedding structures of various black holes

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(Dated: November 29, 2003)

We study global flat embeddings inside and outside of event horizons of black holes such as Schwarzschild and Reissner-Nordström black holes, and of de Sitter space. On these overall patches of the curved manifolds we investigate four accelerations and Hawking temperatures by introducing relevant Killing vectors.

PACS numbers: 02.40.Vh, 04.20.Jb, 04.50.+h, 04.62.+v, 04.70.Dy

Keywords: de Sitter space; Reissner-Nordström black hole; global embedding; Hawking temperature

I. INTRODUCTION

Ever since the discovery that thermodynamic properties of black holes in anti-de Sitter (AdS) spacetime are dual to those of a field theory in one dimension fewer [1], there has been of much interest in the Reissner-Nordström (RN) black hole [2], which now becomes a prototype example for studying this AdS/CFT correspondence [3]. It is also well understood that, in differential geometry four dimensional Schwarzschild metric [4] is not embedded in R^5 [5]. Moreover, n dimensional spacetime has been shown to be embedded into d dimensional pseudo-Euclidean space with dimensionality $n \leq d \leq n(n+1)/2$ [6], so that more than ten dimensions cannot be required to embed any four dimensional solution of Einstein equations with arbitrary energy-momentum tensor.

Recently, (5+1) dimensional global embedding Minkowski space (GEMS) structure for the region outside the event horizon of the Schwarzschild black hole has been obtained [7, 8] to investigate a thermal Hawking effect on a curved manifold [9] associated with an Unruh effect [10] in these higher dimensional space time where the usual black hole detectors are mapped into Rindler observers with the correct temperatures as determined from their constant accelerations. The multiply warped product manifold associated with the (3+1) RN metric has been also studied to investigate the geometrical properties “inside” the event horizons [11]. In this analysis, all the expressions of the Ricci components and the Einstein scalar curvature were shown to be form invariant both in the exterior and interior of the outer event horizon without discontinuities. It has been also shown in the GEMS approach to the (2+1) dimensional black holes that the uncharged and charged Banados-Teitelboim-Zanelli (BTZ) black holes [12, 13] are embedded in (2+2) [7, 14] and (3+2) dimensions [15], while the uncharged and charged black strings are embedded in (3+1) and (3+2) dimensions [16], respectively. Note that the dual solutions of the BTZ black holes are related to the solutions in the string theory, so-called (2+1) black strings [17, 18]. Moreover, in the warped product approach to the BTZ black hole, all the Ricci components and the Einstein scalar curvature have the form invariant expressions both inside and outside the outer event horizon without discontinuities [19]. Quite recently, the BTZ black holes were further analyzed to yield the complete GEMS structures “outside and inside” event horizons [15].

On the other hand, exploiting the Kruskal extension of the Schwarzschild black hole, the coordinate singularity at the event horizon $r = 2m$ can be eliminated to yield in (3+1) dimensional spacetime [20]

$$ds^2 = \frac{32m^3 e^{-r/2m}}{r} (dT^2 - dX^2) - r^2 (d\theta^2 + \sin^2 \theta d\phi^2), \quad (1.1)$$

via coordinate transformations

$$T = \frac{1}{2} \left(e^{(t+r_*)/4m} - e^{-(t-r_*)/4m} \right), \quad X = \frac{1}{2} \left(e^{(t+r_*)/4m} + e^{-(t-r_*)/4m} \right), \quad (1.2)$$

with the Regge-Wheeler tortoise coordinate r_* defined by

$$r_* = r + 2m \ln \left(\frac{r}{2m} - 1 \right). \quad (1.3)$$

Note that the metric (1.1) is not a flat metric and the Kruskal coordinates (1.2) are inconvenient for analyzing the asymptotically flat region $r \rightarrow \infty$ [21]. However, the higher dimensional flat embedding of the Schwarzschild black

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hole, which will be discussed later, is well-defined in the asymptotically flat region [8]. Even though the region outside the event horizon of the Schwarzschild black hole has been nicely described in this GEMS structure to investigate the thermodynamic properties of black hole in terms of the Unruh effect, the region inside the event horizon remains intact with a brief comment that the extension to the interior region is just the maximal Kruskal one [8].

In this paper we will investigate the region “inside” the event horizon of the Schwarzschild black hole to construct explicitly complete embedding solutions, four accelerations and Hawking temperatures inside the event horizons. Moreover, we will construct embedding solutions inside and outside the event horizons of the de Sitter (dS) space and RN black hole to investigate their four accelerations and Hawking temperatures on the overall patches of these curved manifolds. We will consider the Schwarzschild black hole in section 2, the RN black hole in section 3 and the dS spaces in section 4, respectively.

II. SCHWARZSCHILD BLACK HOLE

We begin with a brief recapitulation of the results of the global embedding Minkowski space (GEMS) approach [7], for the (3+1) dimensional Schwarzschild black hole [4] whose four-metric is given by

$$ds^2 = N^2 dt^2 - N^{-2} dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\phi^2), \quad (2.1)$$

where the exterior lapse function is

$$N^2 = 1 - \frac{2m}{r} = \frac{r - r_H}{r}, \quad (2.2)$$

with the event horizon $r_H = 2m$. The (5+1) minimal Schwarzschild GEMS

$$ds^2 = \eta_{MN} dz^M dz^N, \quad \eta_{MN} = \text{diag}(+ - - - -) \quad (2.3)$$

is then given by the coordinate transformations for $r \geq r_H$ as follows [7]

$$\begin{aligned} z^0 &= k_H^{-1} \left(\frac{r - r_H}{r} \right)^{1/2} \sinh k_H t, \\ z^1 &= k_H^{-1} \left(\frac{r - r_H}{r} \right)^{1/2} \cosh k_H t, \\ z^2 &= \int dr \left(\frac{r_H(r^2 + r_H r + r_H^2)}{r^3} \right)^{1/2} \equiv f(r, r_H), \\ z^3 &= r \sin \theta \cos \phi, \\ z^4 &= r \sin \theta \sin \phi, \\ z^5 &= r \cos \theta \end{aligned} \quad (2.4)$$

with the surface gravity $k_H = 1/2r_H$.

Now, to construct the GEMS inside the event horizon $r \leq r_H$, we use the four-metric

$$ds^2 = \bar{N}^{-2} dr^2 - \bar{N}^2 dt^2 - r^2 (d\theta^2 + \sin^2 \theta d\phi^2), \quad (2.5)$$

where the interior lapse function is then given by

$$\bar{N}^2 = -1 + \frac{2m}{r} = \frac{r_H - r}{r}. \quad (2.6)$$

We introduce an ansatz for the coordinate transformations for $r \leq r_H$

$$z^M = (r \cosh k_H t \sin R, r \sinh k_H t \sin R, f, r \sin \theta \cos \phi, r \sin \theta \sin \phi, r \cos \theta) \quad (2.7)$$

where f is defined as in (2.4) and $\sin R$ will be fixed later, to obtain the (5+1) GEMS structure (2.3) for $r \leq r_H$ which yields

$$ds^2 = -\bar{N}^2 dt^2 + (\sin R dr + r \cos R dR)^2 - dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\phi^2) - df^2. \quad (2.8)$$

With the ansatz for $\sin R$ and $\cos R$

$$\sin R = \frac{1}{k_H r} \left(\frac{r_H - r}{r} \right)^{1/2}, \quad \cos R = \frac{1}{k_H r} \left(\frac{k_H^2 r^3 - r_H + r}{r} \right)^{1/2}, \quad (2.9)$$

(2.8) reproduce the Schwarzschild four-metric (2.5) associated with the interior lapse function (2.6), to arrive at the (5+1) GEMS structure (2.3) with the coordinate transformations in the region $r \leq r_H$

$$\begin{aligned} z^0 &= k_H^{-1} \left(\frac{r_H - r}{r} \right)^{1/2} \cosh k_H t, \\ z^1 &= k_H^{-1} \left(\frac{r_H - r}{r} \right)^{1/2} \sinh k_H t, \\ z^2 &= f(r, r_H), \end{aligned} \quad (2.10)$$

with (z^3, z^4, z^5) in (2.4). Here note that the coordinate singularity at $r = r_H$ does not appear in the transformations (2.4) and (2.10). Next, introducing the Killing vector $\xi = \partial_r$ inside the event horizon we obtain the four acceleration

$$a_4 = \frac{r_H}{2r^{3/2}(r_H - r)^{1/2}} \quad (2.11)$$

and the local Hawking temperature inside the event horizon

$$T = \frac{a_4}{2\pi} = \frac{1}{4\pi r_H} \left(\frac{r}{r_H - r} \right)^{1/2}. \quad (2.12)$$

Note that the role of timelike Killing vector $\xi = \partial_t$ defined outside the event horizon is replaced by that of the “timelike” Killing vector $\xi = \partial_r$ in the “interior” Schwarzschild solution associated with the four-metric (2.5). The black hole temperature is then given by

$$T_0 = \frac{1}{4\pi r_H}. \quad (2.13)$$

Next, in order to investigate further the global embedding structure outside the event horizon, we can take the ansatz for z^M

$$z^M = (r \sinh k_H t \sinh R, r \cosh k_H t \sinh R, f, r \sin \theta \cos \phi, r \sin \theta \sin \phi, r \cos \theta) \quad (2.14)$$

with f in (2.4) and $\sinh R$ and $\cosh R$ defined as

$$\sinh R = \frac{1}{k_H r} \left(\frac{r - r_H}{r} \right)^{1/2}, \quad \cosh R = \frac{1}{k_H r} \left(\frac{k_H^2 r^3 + r - r_H}{r} \right)^{1/2}, \quad (2.15)$$

to reproduce the Schwarzschild metric (2.1) associated with the lapse function (2.2) so that we can reconstruct the (5+1) GEMS structure (2.3) with the coordinate transformations (2.4) for $r \geq r_H$. Moreover, introducing the Killing vector $\xi = \partial_t$ outside the event horizon we obtain the four acceleration

$$a_4 = \frac{r_H}{2r^{3/2}(r - r_H)^{1/2}}, \quad (2.16)$$

and the local Hawking temperature [7]

$$T = \frac{1}{4\pi r_H} \left(\frac{r}{r - r_H} \right)^{1/2}. \quad (2.17)$$

Moreover, the black hole temperature T_0 is the same as that inside the event horizon in (2.13).

III. REISSNER-NORDSTRÖM BLACK HOLE

Now, in order to investigate the GEMS structure in the range between the inner and outer event horizons for the nonextremal case of the (3+1) dimensional RN black hole [2], we introduce the four-metric (2.5) with the lapse function

$$\bar{N}^2 = -1 + \frac{2m}{r} - \frac{Q^2}{r^2}. \quad (3.1)$$

Note that two event horizons $r_{\pm}(Q)$ satisfy the equations $0 = -1 + 2m/r_{\pm} - Q^2/r_{\pm}^2$, and the lapse function can be rewritten in terms of these outer and inner horizons

$$\bar{N}^2 = \frac{(r_+ - r)(r - r_-)}{r^2} \quad (3.2)$$

which is well defined for $r_- \leq r \leq r_+$, and the parameters m and Q can be rewritten in terms of r_{\pm} as follows

$$m = \frac{r_+ + r_-}{2}, \quad Q^2 = r_+ r_-. \quad (3.3)$$

Introducing in the region $r_- \leq r \leq r_+$

$$z^M = (r \cosh k_H t \sin R, r \sinh k_H t \sin R, f, r \sin \theta \cos \phi, r \sin \theta \sin \phi, r \cos \theta, g) \quad (3.4)$$

with f and g fixed later and the surface gravity $k_H = (r_+ - r_-)/2r_+^2$, we obtain

$$ds^2 = -\bar{N}^2 dt^2 + (\sin R dr + r \cos R dR)^2 - dr^2 - r^2(d\theta^2 + \sin^2 \theta d\phi^2) - df^2 + dg^2. \quad (3.5)$$

With the ansatz for $\sin R$ and $\cos R$

$$\sin R = \frac{1}{k_H r} \left(\frac{(r_+ - r)(r - r_-)}{r^2} \right)^{1/2}, \quad \cos R = \frac{1}{k_H r} \left(\frac{k_H^2 r^4 - (r_+ - r)(r - r_-)}{r^2} \right)^{1/2}, \quad (3.6)$$

we can construct, after some algebra, the (5+2) minimal RN GEMS structure

$$ds^2 = \eta_{MN} dz^M dz^N, \quad \eta_{MN} = \text{diag}(- - - - - +) \quad (3.7)$$

with the coordinate transformations for $r_- \leq r \leq r_+$

$$\begin{aligned} z^0 &= k_H^{-1} \left(\frac{(r_+ - r)(r - r_-)}{r^2} \right)^{1/2} \cosh k_H t, \\ z^1 &= k_H^{-1} \left(\frac{(r_+ - r)(r - r_-)}{r^2} \right)^{1/2} \sinh k_H t, \\ z^2 &= \int dr \left(\frac{r^2(r_+ + r_-) + r_+^2(r + r_+)}{r^2(r - r_-)} \right)^{1/2} \equiv f(r, r_+, r_-), \\ z^3 &= r \sin \theta \cos \phi, \\ z^4 &= r \sin \theta \sin \phi, \\ z^5 &= r \cos \theta, \\ z^6 &= \int dr \left(\frac{4r_+^5 r_-}{r^4(r_+ - r_-)^2} \right)^{1/2} \equiv g(r, r_+, r_-). \end{aligned} \quad (3.8)$$

Exploiting the Killing vector $\xi = \partial_r$ for $r_- \leq r \leq r_+$, as in the interior Schwarzschild black hole solution case, we obtain the four acceleration

$$a_4 = \frac{(r_+ + r_-)r - 2r_+ r_-}{2r^2[(r_+ - r)(r - r_-)]^{1/2}}, \quad (3.9)$$

the local Hawking temperature

$$T = \frac{a_7}{2\pi} = \frac{(r_+ - r_-)r}{4\pi r_+^2 [(r_+ - r)(r - r_-)]^{1/2}}, \quad (3.10)$$

and the black hole temperature

$$T_0 = \frac{r_+ - r_-}{4\pi r_+^2}. \quad (3.11)$$

Next, for the case of the range inside the inner event horizon $r \leq r_-$, we introduce z^M

$$z^M = (r \sinh k_H t \sinh R, r \cosh k_H t \sinh R, f, r \sin \theta \cos \phi, r \sin \theta \sin \phi, r \cos \theta, g) \quad (3.12)$$

with f and g in (3.8), to arrive at the GEMS structure (3.7) yielding

$$ds^2 = N^2 dt^2 - (\sinh R dr + r \cosh R dR)^2 - dr^2 - r^2(d\theta^2 + \sin^2 \theta d\phi^2) - df^2 + dg^2, \quad (3.13)$$

where the RN lapse function for $r \leq r_-$ is given by

$$N^2 = 1 - \frac{2m}{r} + \frac{Q^2}{r^2} = \frac{(r_+ - r)(r_- - r)}{r^2}. \quad (3.14)$$

With the ansatz for $\sinh R$ and $\cosh R$

$$\sinh R = \frac{1}{k_H r} \left(\frac{(r_+ - r)(r_- - r)}{r^2} \right)^{1/2}, \quad \cosh R = \frac{1}{k_H r} \left(\frac{k_H^2 r^4 + (r_+ - r)(r_- - r)}{r^2} \right)^{1/2}, \quad (3.15)$$

we can produce the four-metric (2.1) associated with the RN lapse function (3.14), and the coordinate transformations of the (5+2) GEMS structure (3.7) for $r \leq r_-$ is given by

$$\begin{aligned} z^0 &= k_H^{-1} \left(\frac{(r_+ - r)(r_- - r)}{r^2} \right)^{1/2} \sinh k_H t, \\ z^1 &= k_H^{-1} \left(\frac{(r_+ - r)(r_- - r)}{r^2} \right)^{1/2} \cosh k_H t, \\ z^2 &= f(r, r_+, r_-), \\ z^6 &= g(r, r_+, r_-), \end{aligned} \quad (3.16)$$

with (z^3, z^4, z^5) and f and g in (3.8). Note that the coordinate singularities at $r = r_{\pm}$ do not appear in the transformation (3.16) as well as in (3.8). Introducing the Killing vector $\xi = \partial_t$ in the region $r \leq r_-$ we obtain the four acceleration

$$a_4 = \frac{(r_+ + r_-)r - 2r_+r_-}{2r^2[(r_+ - r)(r_- - r)]^{1/2}} \quad (3.17)$$

and the local Hawking temperature

$$T = \frac{(r_+ - r_-)r}{4\pi r_+^2 [(r_+ - r)(r_- - r)]^{1/2}}, \quad (3.18)$$

and the black hole temperature T_0 is the same as (3.11) of the region $r_- \leq r \leq r_+$.

Finally, it seems appropriate to comment on the GEMS structure outside the outer event horizon $r \geq r_+$ of the RN black hole. In this region we have the four-metric (2.1) with the lapse function which is the same as (3.14) of the region $r \leq r_-$. Moreover, after some algebra we can obtain the (5+2) GEMS structure (3.7) for $r \geq r_+$ with the same coordinate transformations (3.16) [7, 22]. Introducing the Killing vector $\xi = \partial_t$ again outside the outer event horizon, we can obtain the four acceleration

$$a_4 = \frac{(r_+ + r_-)r - 2r_+r_-}{2r^2[(r - r_+)(r - r_-)]^{1/2}}, \quad (3.19)$$

the local Hawking temperature [7]

$$T = \frac{(r_+ - r_-)r}{4\pi r_+^2 [(r - r_+)(r - r_-)]^{1/2}}, \quad (3.20)$$

and the black hole temperature T_0 is the same as (3.11). Note that a_4 , T and T_0 in this region are the same as those in the region $r \leq r_-$. Moreover, in the limit of $r_- \rightarrow 0$, these quantities a_4 , T and T_0 are reduced to those of the Schwarzschild black hole solution, (2.16), (2.17) and (2.13).

IV. DE SITTER SPACE

In this section, we begin with the (3+1) dimensional dS space described by the four-metric (2.5) with the exterior lapse function

$$\bar{N}^2 = -1 + \frac{r^2}{l^2} = \frac{r^2 - r_H^2}{r_H^2}, \quad (4.1)$$

with the event horizon $r_H = l$ satisfying the equation $0 = -1 + r_H^2/l^2$. In order to construct the GEMS outside the event horizon, we introduce

$$z^M = (\cosh k_H t \sinh R, \sinh k_H t \sinh R, r \sin \theta \cos \phi, r \sin \theta \sin \phi, r \cos \theta) \quad (4.2)$$

with the surface gravity $k_H = 1/r_H$, to obtain for $r \geq r_H$

$$ds^2 = -\bar{N}^2 dt^2 + \cosh^2 R dR^2 - dr^2 - r^2(d\theta^2 + \sin^2 \theta d\phi^2). \quad (4.3)$$

With the ansatz for $\sinh R$ and $\cosh R$,

$$\sinh R = (r^2 - r_H^2)^{1/2}, \quad \cosh R = (1 + r^2 - r_H^2)^{1/2}, \quad (4.4)$$

the four-metric (4.3) yields the (4+1) GEMS structure for $r \geq r_H$

$$ds^2 = \eta_{MN} dz^M dz^N, \quad \eta_{MN} = \text{diag}(+ - - -), \quad (4.5)$$

with the coordinate transformations

$$\begin{aligned} z^0 &= k_H^{-1} \left(\frac{r^2 - r_H^2}{r_H^2} \right)^{1/2} \cosh k_H t, \\ z^1 &= k_H^{-1} \left(\frac{r^2 - r_H^2}{r_H^2} \right)^{1/2} \sinh k_H t, \\ z^2 &= r \sin \theta \cos \phi, \\ z^3 &= r \sin \theta \sin \phi, \\ z^4 &= r \cos \theta. \end{aligned} \quad (4.6)$$

Introducing the Killing vector $\xi = \partial_r$ outside the event horizon as in the previous sections, we obtain the four acceleration

$$a_4 = \frac{r}{r_H (r^2 - r_H^2)^{1/2}}, \quad (4.7)$$

the local Hawking temperature

$$T = \frac{a_5}{2\pi} = \frac{1}{2\pi (r^2 - r_H^2)^{1/2}}, \quad (4.8)$$

and the temperature T_0 given by

$$T_0 = \frac{1}{2\pi r_H}. \quad (4.9)$$

Next, for the case of the range inside the event horizon $r \leq r_H$ where we have the four-metric (2.1) with the interior lapse function

$$N^2 = 1 - \frac{r^2}{l^2} = \frac{r_H^2 - r^2}{r_H^2}, \quad (4.10)$$

we introduce z^M

$$z^M = (\sinh k_H t \sin R, \cosh k_H t \sin R, r \sin \theta \cos \phi, r \sin \theta \sin \phi, r \cos \theta) \quad (4.11)$$

to yield

$$ds^2 = N^2 dt^2 - \cos^2 R dR^2 - dr^2 - r^2(d\theta^2 + \sin^2 \theta d\phi^2). \quad (4.12)$$

Now we can readily check that (4.12) can be satisfied with the ansatz for $\sin R$ and $\cos R$

$$\sin R = (r_H^2 - r^2)^{1/2}, \quad \cos R = (1 - r_H^2 + r^2)^{1/2}, \quad (4.13)$$

to produce the (4+1) GEMS structure (4.5) for $r \leq r_H$ associated with the coordinate transformations [7]

$$\begin{aligned} z^0 &= k_H^{-1} \left(\frac{r_H^2 - r^2}{r_H^2} \right)^{1/2} \sinh k_H t, \\ z^1 &= k_H^{-1} \left(\frac{r_H^2 - r^2}{r_H^2} \right)^{1/2} \cosh k_H t, \end{aligned} \quad (4.14)$$

with (z^2, z^3, z^4) in (4.6). Note that the coordinate singularity at $r = r_H$ does not appear any more in the transformation (4.14) and (4.6), in which we can readily find the identity $\eta_{MN} z^M z^N = -l^2$. Introducing the Killing vector $\xi = \partial_t$ we also obtain the four acceleration

$$a_4 = \frac{r}{r_H(r_H^2 - r^2)^{1/2}} \quad (4.15)$$

and the local Hawking temperature [7]

$$T = \frac{1}{2\pi(r_H^2 - r^2)^{1/2}}. \quad (4.16)$$

Moreover, the temperature T_0 is the same as that outside the event horizon given in (4.9).

V. CONCLUSIONS

In conclusion, we have constructed the complete embedding solutions, four accelerations and Hawking temperatures inside and outside the event horizons of the dS space, Schwarzschild and RN black holes to explicitly calculate four accelerations and Hawking temperatures on the overall patches of these curved manifolds by introducing the relevant Killing vectors. It was shown in these manifolds that the temperatures T_0 are identical on these overall patches, while the four accelerations a_4 and local Hawking temperatures T have different expressions dependent on the interiors and exteriors of the event horizons.

Acknowledgments

The author acknowledges financial support from the Korea Science and Engineering Foundation Grant R01-2000-00015.

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